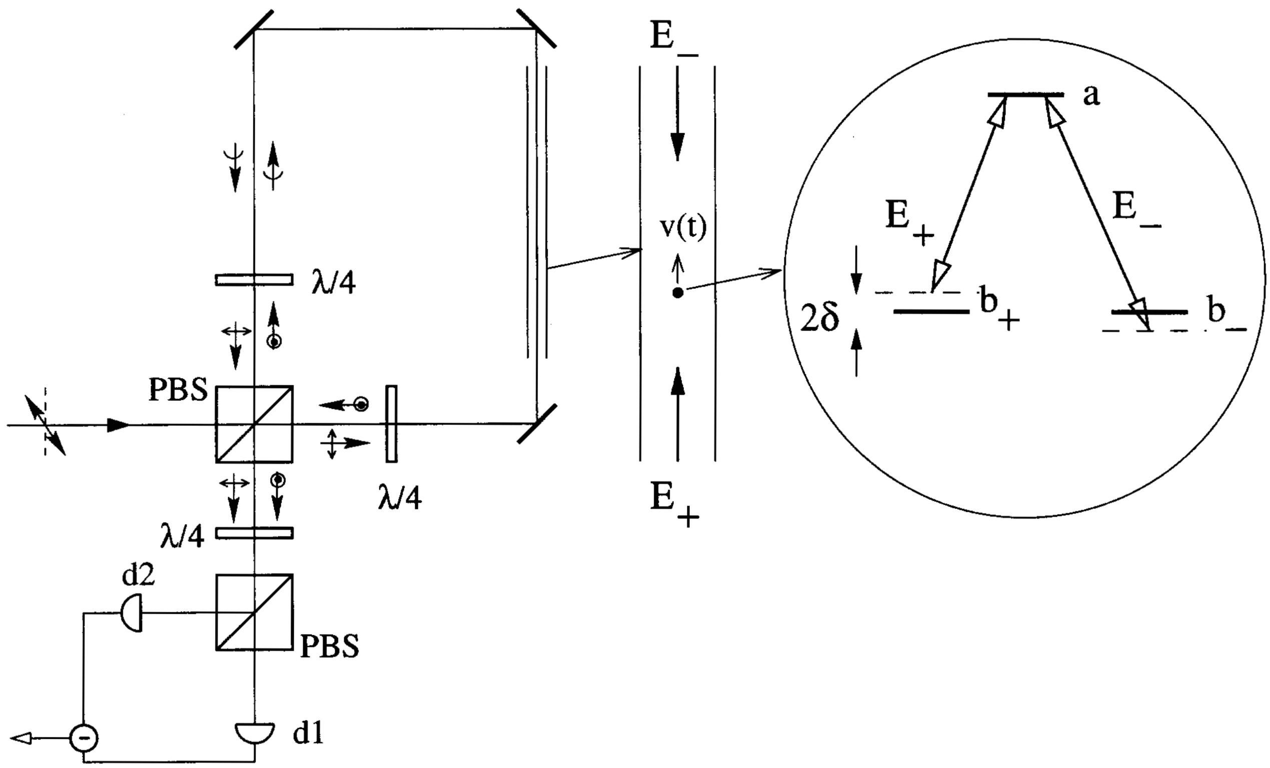
Optical measurements of gravity fields

Andrey Matsko, Nan Yu, and Lute Maleki

Jet Propulsion Laboratory, CalTech

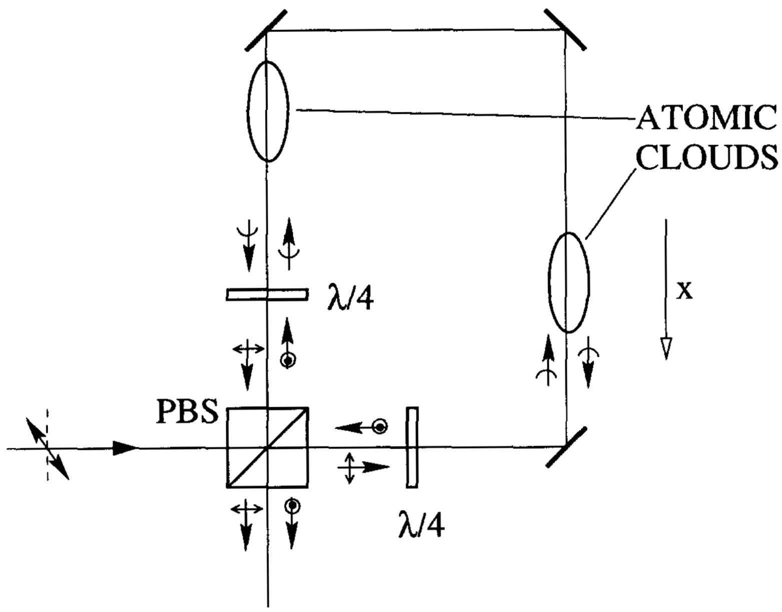
1. Optical measurements of a gravitational field with sensitivity close to the sensitivity of atomic devices are possible if one detects properties of light after its interaction with optically thick atomic cloud moving freely in the gravity field.
2. A nondestructive detection of a number of ultracold atoms in a cloud as well as tracking of the ground state population distribution of the atoms is possible by optical means.

Gravity field detection: measurement scheme



A three-level atom interacts with two counterpropagating waves having the opposite circular polarizations. The atomic motion results in detuning of the fields from the corresponding atomic transitions in atom's frame of reference. This detuning results in change of the index of refraction for the waves that may be measured in the interferometric scheme: photocurrents are $I_{d1,d2} \sim (1/2)\left[P_+ + P_- \pm 2\sqrt{P_+ P_-}\sin\ \phi(\delta)\right]$.

Gravity gradient detection: measurement scheme



Two clouds of three-level atoms interacts with two counterpropagating waves having the opposite circular polarizations. Gravity field gradient changes atomic velocity and this results in change of the relative phase of the electromagnetic waves.

Interaction Hamiltonian

Hamiltonian

$$\tilde{H} = \frac{P^2}{2m} + mgx + \hbar\Delta\hat{\sigma}_{aa} - \\ (\hbar\Omega_+\hat{\sigma}_{b+a}e^{ik_+x} + \hbar\Omega_-\hat{\sigma}_{b-a}e^{-ik_-x} - adj.),$$

may be transformed to

$$H = \frac{1}{2m}\left[P + \frac{\hbar\omega_0}{c}(\hat{\sigma}_{b+b+} - \hat{\sigma}_{b-b-})\right]^2 + mgx + \\ \hbar\Delta\hat{\sigma}_{aa} - (\hbar\Omega_+\hat{\sigma}_{b+a}e^{ik_+x} + \hbar\Omega_-\hat{\sigma}_{b-a}e^{-ik_-x} - adj.),$$

where

$$P = mv - \frac{\hbar\omega_0}{c}(\hat{\sigma}_{b+b+} - \hat{\sigma}_{b-b-}).$$

is the generalized atomic momentum. P is a QND observable. Equation of motion for an atom is

$$\dot{P} = -mg \rightarrow P(t) = P(0) - mgt.$$

Two-photon detuning:

$$\delta = \frac{k}{m}(P(0) - mgt).$$

Propagation problem

Maxwell equations:

$$\frac{\mathrm{d}}{\mathrm{d}x}E_{\pm}(x) = \pm\frac{2i\pi\omega_0}{c}\wp_{\pm}N\sigma_{ab\pm}(x).$$

Solution for $\phi = \phi_+(L) - \phi_-(0)$:

$$\phi = \kappa\int_0^L \frac{\gamma_{\mathrm{r}}\delta}{|\Omega|^2}dx = -\frac{\delta}{\gamma_0}\ln\left[1 - \kappa L\frac{\gamma_0\gamma_{\mathrm{r}}}{|\Omega_0|^2}\right] =$$

$$-\frac{\omega_0}{\gamma_0}\frac{P(0) - mgt}{mc}\ \ln\eta,$$

where

$$\eta = \frac{P_{out}}{P_{in}} = 1 - \frac{3}{4\pi}\lambda^2 NL\frac{\gamma_0\gamma_r}{|\Omega_0|^2}.$$

Filtration procedure

The difference photocurrent is

$$I_- \sim i(E_+^\dagger E_- - E_-^\dagger E_+).$$

We present field E as a sum of an expectation value part and quantum fluctuation part $E = \langle E \rangle + e$, then $I_- = I_S + I_{N1} + I_{N2}$, where

$$\begin{aligned} I_S &\sim 2|\langle E_+\rangle||\langle E_-\rangle| \frac{\omega_0}{\gamma_0}\frac{gt}{c} \ln\eta, \\ I_{N1} &\sim 2|\langle E_+\rangle||\langle E_-\rangle| \frac{\omega_0}{\gamma_0}\frac{P(0)}{mc} \ln\eta, \\ I_{N2} &\sim i|\langle E_-\rangle|(e_+^\dagger - e_+) + i|\langle E_+\rangle|(e_- - e_-^\dagger). \end{aligned}$$

Filtration procedure

$$\widetilde{I}_- = \int_0^T G(t) I_-(t) dt.$$

Signal to noise ratio

$$\frac{S}{N} = \frac{\langle \widetilde{I}_- \rangle}{\sqrt{\langle \widetilde{I}_-^2 \rangle - \langle \widetilde{I}_- \rangle^2}}$$

Sensitivity

We vary $G(t)$ and look for $(S/N)_{max}$.

An approximation from the bottom when initially atoms are incoherent:

$$\left(\frac{S}{N}\right)_{max} \geq \frac{\pi g T^2}{\sqrt{3}\lambda}\sqrt{\mathcal{N}},$$

or coherently prepared in a dark state

$$\left(\frac{S}{N}\right)_{max} = \frac{\pi g T^2}{\sqrt{3}\lambda}\sqrt{\mathcal{N}}\sqrt{\frac{\mathcal{N}}{n_{in}}}.$$

A practical problem: very small optimum photon number: $\mathcal{N} \geq n_{in}$.

Solution: usage of short pulses, instead of quasi CW fields.

Measurement of atomic velocity

Let us consider an atom initially prepared in $|b_+\rangle$ state: $\hat{\sigma}_{b+b+}(0) = 1$. Then $P = mv(0) - \hbar k$.

$P(t) = P(0)$ if there is no gravity field ($g = 0$).

Quantum Nondemolition Measurements of P and, hence, $v(0)$ are possible. The main requirement: $\hat{\sigma}_{b+b+}(0) = \hat{\sigma}_{b+b+}(T)$, i.e. there is no spontaneous emission.

Maximal measurement sensitivity:

$$v(0)_{min} \simeq \frac{c}{\omega_0 T}.$$

Note: Energy

$$\Delta\mathcal{E} = \frac{mv_{min}^2}{2} = \frac{mc^2}{2\omega_0^2 T^2},$$

measurement time $\Delta t = T$, and there exists a possibility that

$$\Delta\mathcal{E}\Delta t = \frac{mc^2}{2\omega_0^2 T} < \hbar.$$

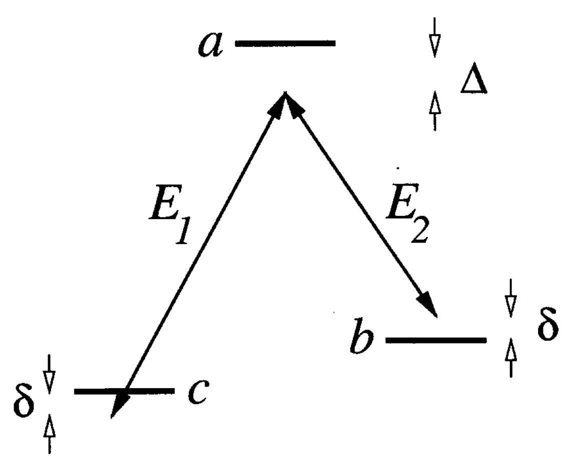
Experiments with pulsed laser

Applications:

- Measurements of gravity field
- Nondestructive optical detection of ultracold atoms

Main differences from the previous scheme:

- Off-resonant (Raman) configuration
- Short pulses instead of quasi-CW light
- Information is saved in pulse photon numbers, not in the phase or polarization



A three-level atom interacts with two counter propagating waves. The atomic motion results in detuning δ of the fields from the corresponding atomic transitions in atom's frame of reference. One photon detuning Δ is much larger than γ_a.

Interaction Hamiltonian and measurement strategy

$$\begin{aligned}\tilde{H}/\hbar &= \left(\delta - \frac{|\Omega_1|^2 - |\Omega_2|^2}{\Delta}\right)(\hat{\sigma}_b - \hat{\sigma}_c) \\ &+ |b\rangle\langle c|\frac{\Omega_2^\dagger\Omega_1}{\Delta} + |c\rangle\langle b|\frac{\Omega_1^\dagger\Omega_2}{\Delta}.\end{aligned}$$

Photon number conservation law:

$$\dot{\hat{n}}_1 = \dot{\hat{\sigma}}_c = -\dot{\hat{\sigma}}_b = -\dot{\hat{n}}_2.$$

Therefore, if atomic population changes, the photon numbers change as well.

We consider a sequence of $\pi/2 - \pi - \pi/2$ pulses. If the atom is initially in the state $|b\rangle$ field E_2 of the first $\pi/2$ Raman pulse looses $\mathcal{N}/2$ photons after the interaction, field E_1, in turn, gains $\mathcal{N}/2$ photons $\rightarrow$ Measurement of $\mathcal{N}$ with accuracy $\sqrt{n}$.

Detection of photon number in the second $\pi/2$ gives information about the signal. The measurement sensitivity is

$$\frac{S}{N} = \frac{2\frac{\omega_0}{c}gT^2\sqrt{\mathcal{N}}}{\sqrt{1+\frac{4n}{\mathcal{N}}}}.$$

Conclusion

- In conclusion, we propose a method of detection of gravity field gradients using polarization spectroscopy. Atomic motion causes the polarization rotation that carry information about atomic acceleration. Optimum filtration results in measurement sensitivity comparable with the sensitivity of already existing methods of atomic interferometry.

- We have shown that measurements of absorption of light pulses that are used in conventional atomic interferometry for the preparation of atomic clouds may provide information about number of cold atoms participating in the interaction. This will improve the sensitivity of existing atomic interferometric schemes without introduction of significant changes to them. Moreover, the measurement of the light pulses may also give a rough estimation for the expected interferometric signal.